List all the factors of each number in the table below. (DOK 1)

Number	Factors
8	
20	
16	
40	
12	
32	

Answer the following questions. (DOK 1)

1. What is the GCF of 16 and 20?

2. What is the GCF of 32 and 40?

3. What is the GCF of 6 and 8?

4. What is the GCF of 9 and 12?

5. What is the GCF of 10 and 25?

6. What is the GCF of 20 and 8?

7. What is the GCF of 18 and 27?

8. What is the GCF of 40 and 70?

9. What is the GCF of 14 and 35?

10. What is the GCF of 8, 16, and 32?

11. Explain how you can be sure you have found the **greatest** common factor between two numbers and not just any common factor.

Least Common Multiple (DOK 1)

To find the **least common multiple (LCM)** of two numbers, first list the multiples of each number. The multiples of a number are 1 times the number, 2 times the number, 3 times the number, and so on.

The multiples of 6 are: 6, 12, 18, 24, 30, ...

The multiples of 10 are: 10, 20, 30, 40, 50, ...

What is the smallest multiple they both have in common? 30

30 is the least (smallest number) **common multiple** of 6 and 10.

Find the least common multiple (LCM) of each pair of numbers below. (DOK 1)

	Pairs	Multiples	LCM		Pairs	Multiples	LCM
1.	6	6, 12, 18, 24, 30	30	10.	5		
	5	5, 10, 15, 20, 25, 30			7		
2.	12			11.	4		
	6				8		
3.	8			12.	2		
	6				5		
4.	7			13.	3		
	3				4		
5.	12			14.	3		
	8				8		
6.	6			15.	12		
	7				9		
7.	4			16.	5		
	10				11		
8.	9			17.	3		
	6				5		
9.	2			18.	4		
	11				7		

Finding LCM and GCF Together (DOK 2)

Example: Find the GCF and LCM of 16 and 20.

Step 1:   Take the two numbers and write them like below.
$\underline{|16 \qquad 20}$

Step 2:   Find the number that will divide evenly into both numbers.
$2\underline{|16 \qquad 20}$
$\quad 8 \qquad 10$

Step 3:   Repeat the process until you cannot find a common number to divide both numbers by.
$2\underline{|16 \qquad 20}$
$\quad 2\underline{|8 \qquad 10}$
$\qquad 4 \qquad 5$

Step 4:   To find the GCF, multiply the number on the left:
$2 \times 2 = 4$

Step 5:   To find the LCM, multiply all the numbers on the outside:
$2 \times 2 \times 4 \times 5 = 80$

Reciprocals (DOK 1)

When you divide fractions, you change the second number to its reciprocal. This means flipping the fraction.

Example 1:   Find the reciprocal of $\dfrac{3}{8}$

    Step 1:   Invert the fraction, so the numerator is now the denominator, and the denominator is now the numerator.

        $\dfrac{3}{8}$ becomes $\dfrac{8}{3}$.

    Answer: The reciprocal of $\dfrac{3}{8}$ is $\dfrac{8}{3}$.

Example 2:   Find the reciprocal of 14.

    Step 1:   Turn the whole number 14 into a fraction by putting 14 over 1.

        14 becomes $\dfrac{14}{1}$.

    Step 2:   Invert the fraction, so the numerator is now the denominator, and the denominator is now the numerator.

        $\dfrac{14}{1}$ becomes $\dfrac{1}{14}$.

    Answer: The reciprocal of 14 is $\dfrac{1}{14}$.

Find the reciprocal of each fraction below. (DOK 1)

1. $\dfrac{1}{5}$	7. $\dfrac{8}{3}$	13. $\dfrac{3}{5}$	19. $\dfrac{358}{411}$
2. $\dfrac{2}{3}$	8. $\dfrac{5}{17}$	14. $\dfrac{19}{33}$	20. $\dfrac{91}{100}$
3. $\dfrac{5}{8}$	9. $\dfrac{2}{11}$	15. $\dfrac{4}{7}$	21. 18
4. $\dfrac{2}{7}$	10. $\dfrac{15}{21}$	16. $\dfrac{5}{17}$	22. 20
5. $\dfrac{3}{19}$	11. $\dfrac{24}{11}$	17. $\dfrac{124}{127}$	23. 9
6. $\dfrac{4}{13}$	12. $\dfrac{18}{19}$	18. $\dfrac{2}{9}$	24. 143

Dividing Fractions and Mixed Numbers (DOK 3)

One way to remember how to divide with fractions is using the phrase, "keep it, change it, and flip it." Keep the first fraction the same, change the division sign to a multiplication sign, and flip the second fraction (change it to its reciprocal).

Example: Divide: $3\frac{3}{4} \div 2\frac{1}{3}$

Step 1: Change the mixed numbers in the problem to improper fractions.

$3\frac{3}{4} = \frac{(4\times3)+3}{4} = \frac{15}{4}$ and $2\frac{1}{3} = \frac{(3\times2)+1}{3} = \frac{7}{3}$

The problem is now $\frac{15}{4} \div \frac{7}{3}$.

Step 2: Change the sign from division to multiplication, and then change the second number to its reciprocal. $\frac{15}{4} \times \frac{3}{7}$

Step 3: Cancel if possible (not needed in this problem) and multiply.

$\frac{15}{4} \times \frac{3}{7} = \frac{45}{28}$.

Step 4: Simplify to $1\frac{17}{28}$.

Divide and simplify answers to lowest terms. Show your work for each step. (DOK 3)

1. $6\frac{1}{3} \div 2\frac{1}{2}$

7. $70\frac{1}{4} \div 6\frac{1}{2}$

13. $6\frac{7}{8} \div 4\frac{1}{2}$

19. $24\frac{1}{2} \div 3\frac{3}{4}$

2. $8\frac{1}{4} \div 1\frac{1}{2}$

8. $3\frac{3}{7} \div 1\frac{1}{2}$

14. $3\frac{1}{3} \div \frac{7}{9}$

20. $12\frac{1}{3} \div 4\frac{1}{2}$

3. $11\frac{1}{3} \div 2$

9. $5\frac{1}{4} \div 3\frac{1}{2}$

15. $2\frac{2}{5} \div 6\frac{1}{2}$

21. $6\frac{1}{4} \div 2\frac{3}{5}$

4. $15\frac{1}{2} \div \frac{1}{2}$

10. $7 \div 4\frac{1}{2}$

16. $11\frac{1}{2} \div 2\frac{3}{4}$

22. $5\frac{3}{4} \div \frac{2}{5}$

5. $\frac{5}{6} \div \frac{3}{4}$

11. $16\frac{1}{3} \div 4$

17. $51 \div 2\frac{1}{2}$

23. $7\frac{1}{2} \div 3\frac{1}{2}$

6. $18\frac{3}{5} \div 2\frac{1}{5}$

12. $8\frac{2}{5} \div 1\frac{1}{2}$

18. $16\frac{1}{4} \div 2\frac{1}{2}$

24. $4\frac{2}{3} \div 7$

Multiplying Decimals (DOK 2)

Example 3:	56.2×0.17	
Step 1:	Set up the problem as if you were multiplying whole numbers.	$\begin{array}{r} 56.2 \\ \times\ 0.17 \\ \hline \end{array}$
Step 2:	Multiply as if you were multiplying whole numbers.	$\begin{array}{r} 41 \\ 56.2 \\ \times\ 0.17 \\ \hline 3934 \\ 562\ \ \\ \hline 9.554 \end{array}$

1 numbers after the decimal point
+2 numbers after the decimal point
3 total numbers after the decimal point

Step 3:   Count how many numbers are after the decimal points in the problem. In this problem, 2, 1, and 7 come after decimal points, so the answer must also have three numbers after the decimal point.

Multiply. (DOK 2)

1. 15.2×3.58	5. 45.8×2.29	9. 23.65×9.29	13. $(16.4)(0.599)$
2. 9.54×5.32	6. 4.59×7.18	10. 1.54×0.432	14. $(0.87)(3.21)$
3. 5.72×6.3	7. 0.052×0.33	11. $(0.47)(6.19)$	15. $(5.94)(0.65)$
4. 4.81×3.27	8. 4.12×68.7	12. $(1.3)(1.57)$	16. $(7.8)(0.23)$

Dividing Decimals (DOK 2)

Example: $374.5 \div 0.07$

Step 1:   Copy the problem as you would for whole numbers.

Divisor

$0.07\overline{)374.5}$ ← **Dividend**

Step 2:   You cannot divide by a decimal number. You must get rid of the decimal in the divisor. You must shift the decimal in the divisor to the right until you get a whole number. What you do to the divisor, you must do to the dividend. Shift the dividend decimal to the right the same number of places as you shifted the divisor decimal.

$0.07.\overline{)374.50.}$

Step 3:   The problem now becomes $37450 \div 7$. Copy the decimal point from the dividend straight above in the place for the answer.

$$\begin{array}{r} 5350. \\ 7{\overline{)37450.}} \\ -35\ \ \ \ \ \\ \hline 24\ \ \ \\ -21\ \ \ \\ \hline 35\ \\ -35\ \\ \hline 00 \end{array}$$

Divide. Remember to move the decimal points. (DOK 2)

1. $0.676 \div 0.013$
2. $70.32 \div 0.08$
3. $\$54.60 \div 0.84$
4. $\$10.35 \div 0.45$

5. $18.46 \div 1.3$
6. $14.6 \div 0.002$
7. $\$125.25 \div 0.75$
8. $\$33.00 \div 1.65$

9. $154.08 \div 1.8$
10. $0.4374 \div 0.003$
11. $292.9 \div 0.29$
12. $6.375 \div 0.3$

13. $4.8 \div 0.08$
14. $1.2 \div 0.024$
15. $15.725 \div 3.7$
16. $\$167.50 \div 0.25$

Evaluating Decimal Equivalence (DOK 1)

Place the expressions in the correct column for each table. (DOK 1)

1.

Equal to 9.32	Not Equal to 9.32

$9.01 + 31$ 2×4.66 $14.52 - 5.1$
$27.96 \div 3$ $8.01 + 1.31$ 4×2.08

2.

Equal to 30.36	Not Equal to 30.36

$60.72 \div 2$ 6×5.06 $40.46 - 9.6$
$30 + 36$ $30.5 + 0.6$ 6×6.06

3.

Equal to 44.6	Not Equal to 44.6

$88.6 \div 2$ 11.15×4 $40 + 6.4$
$89.2 \div 2$ $34 + 10.6$ 2×22.3

4.

Equal to 72.24	Not Equal to 72.24

24.08×3 $72.2 + 4$ 18.06×4
$77.24 - 5$ $36.12 + 36$ $72 + 24$

5.

Equal to 6.81	Not Equal to 6.81

2.27×3 $3.81 + 2$ $34.05 \div 5$
$6.81 \div 1$ $12.81 \div 2$ $2.41 + 4.4$

6.

Equal to 22.44	Not Equal to 22.44

$12 + 1.14$ 4×5.61 22.44×0
11.22×2 $67.32 \div 3$ $31 - 9.6$

Example 2:   Compare $\frac{4}{5}$ _____ $\frac{7}{10}$ using $>, <,$ or $=$.

Step 1:   Compare the denominators by finding the least common multiple. The LCM of 5 and 10 is 10.

Step 2:   Change the fractions, so they have the same denominator.

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

Step 3:   Compare $\frac{8}{10} > \frac{7}{10}$.

Therefore, $\frac{4}{5} > \frac{7}{10}$.

To compare decimals:

1. Write the decimal numbers in a vertical column or make a table. Make sure the decimal points are in line.
2. Compare the numbers in the ones (units) place.
3. Compare the numbers in the tenths place.
4. Continue comparing each 'place' until all digits have been ordered.

Example 3:   A student's average is 89.31. Another student's average is 89.21. Which student has the larger average?

Step 1:   Compare each number in each place value to find the difference. There is a difference in the tenths place. $3 > 2$

Answer: 89.31 is greater than 89.21. $89.31 > 89.21$

Fill in the box with the correct sign ($>, <,$ or $=$). (DOK 2)

1. $-1 \Box -6$

2. $-18 \Box -7$

3. $\frac{4}{6} \Box -\frac{7}{12}$

4. $-\frac{3}{4} \Box \frac{-1}{3}$

5. $-\frac{5}{8} \Box -1$

6. $-62 \Box -61$

7. $\frac{9}{10} \Box \frac{2}{5}$

8. $-\frac{1}{10} \Box -1$

9. $-47 \Box -47\frac{1}{2}$

10. $-\frac{1}{3} \Box -\frac{1}{2}$

11. $-\frac{1}{12} \Box -\frac{1}{3}$

12. $-5 \Box -50$

13. $-\frac{8}{9} \Box -\frac{2}{3}$

14. $-4 \Box -3\frac{1}{2}$

15. $-\frac{10}{9} \Box -\frac{9}{10}$

16. $-100 \Box -101$

17. $-990 \Box -999$

18. $-\frac{5}{6} \Box -\frac{1}{6}$

19. $0.203 \Box 0.23$

20. $0.56 \Box 1.06$

21. $1.34 \Box 1.31$

22. $0.467 \Box 0.4670$

Absolute Value (DOK 2)

The **absolute value** of a number is the distance the number is from zero on the number line.

The absolute value of 6 is written $|6|$. $|6| = 6$

The absolute value of -6 is written $|-6|$. $|-6| = 6$

The absolute value of $-|-6|$ or $-|6|$ is -6 because the negative sign is on the outside of the absolute value sign.

Both 6 and -6 are the same distance, 6 spaces from zero, so their absolute value is the same: 6.

Examples:	
$|-4| = 4$	$|6| - |-6| = 6 - 6 = 0$
$|9| - |8| = 9 - 8 = 1$	$|-9| + 5 = 9 + 5 = 14$
$-|-4| = -4$	$|-5| + |-2| = 5 + 2 = 7$

Simplify the following absolute value problems. (DOK 2)

1. $|9| \ =$

2. $-|5| \ =$

3. $|-25| \ =$

4. $-|-12| \ =$

5. $-|64| \ =$

6. $|-2| \ =$

7. $-|-3| \ =$

8. $|-4| - |3| \ =$

9. $|-8| - |-4| \ =$

10. $|5| + |-4| \ =$

11. $|-2| + |6| \ =$

12. $|10| + |8| \ =$

13. $|-2| + |4| \ =$

14. $|-3| + |-4| \ =$

15. $|7| - |-5| \ =$

Absolute Value Word Problems (DOK 2)

Absolute value is used in the world to describe forms of magnitude, such as distance or debt.

Example 1:	The depth of a sunken ship is 1,950 feet below sea level. This depth can also be expressed as $|-1,950|$ feet.
Example 2:	The day starts out at $54°$, rises to $84°$, and then goes back to $54°$ at the end of the day. The range in temperatures is the highest temperature minus the lowest temperature and is expressed as an absolute value: $|30°|$.

Fill in the ordered pair for each point, and tell which quadrant it is in. (DOK 1)
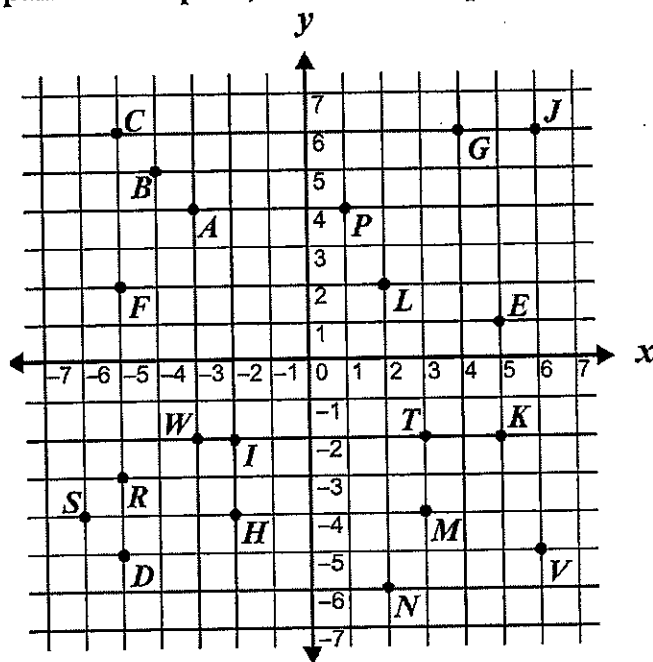
1. point A = (,   )   quadrant____

2. point B = (,   )   quadrant____

3. point C = (,   )   quadrant____

4. point D = (,   )   quadrant____

5. point E = (,   )   quadrant____

6. point F = (,   )   quadrant____

7. point G = (,   )   quadrant____

8. point H = (,   )   quadrant____

9. point I = (,   )   quadrant____

10. point J = (,   )   quadrant____

11. point K = (,   )   quadrant____

12. point L = (,   )   quadrant____

13. point M = (,   )   quadrant____

14. point N = (,   )   quadrant____

15. point P = (,   )   quadrant____

16. point R = (,   )   quadrant____

17. point S = (,   )   quadrant____

18. point T = (,   )   quadrant____

19. point V = (,   )   quadrant____

20. point W = (,   )   quadrant____

Sometimes, points on a coordinate plane fall on the x or y axis. If a point falls on the x-axis, then the second number of the ordered pair is 0. If a point falls on the y-axis, the first number of the ordered pair is 0.

Equivalent Ratios (DOK 1, 2)

Ratios show how two amounts are related. If the ratio of boys to girls in a school is 1:3, this can be written as $\frac{10}{30}$, 2:6, or $\frac{20}{60}$. Each one of these represents an equivalent ratio to 1:3. To find equivalent ratios, write the ratio as a fraction. Then, multiply or divide the top and bottom by the same number. To express the ratio in simplest form, you must divide each number in the ratio by their Greatest Common Factor (GCF).

Example: $\frac{2}{5} = \frac{4}{\square}$

Multiply the 2 numbers diagonal from each other. $5 \times 4 = 20$.
Divide that answer by the other number. $20 \div 2 = 10$.
10 is the missing number.

Use what you know about ratios to solve the following. (DOK 2)

1. Write the ratio $\frac{15}{25}$ in simplest form.
2. Write the ratio 4:8 in simplest form.
3. Write the ratio $\frac{8}{64}$ in simplest form.
4. Write the ratio 14 : 21 in simplest form.

Replace the x, y, and z in the tables of equivalent ratios with the correct number. Do not simplify the answers. (DOK 2)

5.

2 : 3
40 : x
$\frac{y}{6}$
$\frac{12}{z}$

6.

1 : 5
30 : x
$\frac{y}{25}$
$\frac{6}{z}$

7.

3 : 4
21 : x
$\frac{y}{40}$
$\frac{18}{z}$

8.

2 : 7
22 : x
$\frac{y}{28}$
$\frac{4}{z}$

9.

3 : 8
9 : x
$\frac{y}{88}$
$\frac{18}{z}$

10. Mrs. Iachetta's recipe for salad dressing uses a ratio of 1 part olive oil to 2 parts vinegar. If she uses 3 cups olive oil, how much vinegar will she need?

11. Marc's grades are as follows: Reading, 88; Math, 92; Science, 96; Social Studies, 90; and Language Arts, 94. Which classes are compared by the ratio 94:90? (Write them as a ratio.)

12. A map shows a scale of 1 inch = 50 miles. How many miles are represented by 4.5 inches on the map?

Changing Percents to Fractions and Fractions to Percents:

Example 4:   44% written as a fraction is $\frac{44}{100}$, which simplifies to $\frac{11}{25}$. To write a percent as a fraction, the percent number becomes the numerator (top number) of the fraction, and 100 becomes the denominator. Sometimes the fraction needs to be simplified.

Example 5: $\frac{3}{8}$ written as a decimal is $0.375 = 37.5\%$
To change a fraction to a percent, write the fraction as a decimal first. Then move the decimal two spaces to the right and add a percent (%).

Complete the chart. (DOK 2)

	Percent	Fraction	Decimal
1.	15%		
2.			0.39
3.		$\frac{1}{4}$	
4.	35%		
5.			0.99
6.		$\frac{33}{100}$	
7.			0.58
8.	18%		
9.		$\frac{29}{100}$	
10.	67%		

Shade each model to represent the percent indicated. (DOK 2)

11. 58%   12. 18%   13. 79%   14. 99%
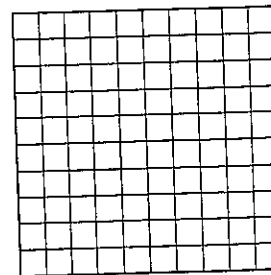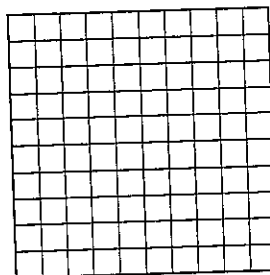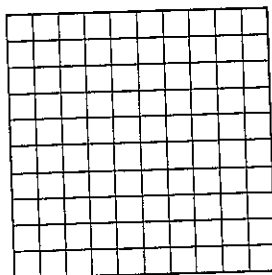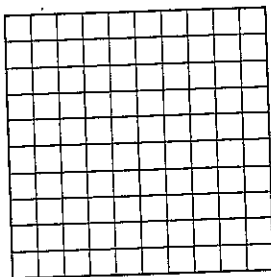
Finding the Whole in a Percent Problem (DOK 2)

There are times when you may be given the part of a whole and the percent of the part, but the whole number is unknown.

Example: Three bottles of water represent 12.5% of a case of bottles of water. How many bottles are there in one whole case?

Step 1:   Set up the following proportion,

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

Step 2:   Find the whole. Use the proportion in step 1.

$$\frac{3}{x} = \frac{12.5}{100}$$

Step 3:   Cross multiply to solve for x:

$3 \times 100 = 300$ ◄————— Multiply the diagonal numbers.

$300 \div 12.5 = 24$ ◄—— Divide by the other number.

$24 = x$

Answer: There are 24 bottles in the case.

Find the whole number in each problem. (DOK 2)

1. Carlos works in a clothing store. Ten hours represent 25% of Carlos' work week. How many hours does Carlos work in one week?

2. Mr. O'Grady harvested his garden. He put the tomatoes into boxes, each box representing 20% of his tomato harvest. Each box holds 24 tomatoes. How many tomatoes did Mr. O'Grady harvest?

3. Fabiola found that she has already used 60 pieces, which is 40% of her notebook paper. How many pieces of paper were in the notebook at the start?

4. In Irene's neighborhood, she found that 12 houses, which is 16%, were painted brown. How many houses are in Irene's neighborhood?

5. Dakota has $90 saved up. Each week he puts 50% of his allowance in his savings account. How much money did Dakota receive in all that he could save $90?

6. Alejandro found that his bag of jelly beans includes 30 red jelly beans, which represent 15% of the entire bag. How many jelly beans are in the whole bag?

7. Warren poured out a cylinder of potato chips. He found that 6 of the chips, which represents 8%, were broken. How many potato chips were there in the whole cylinder?

8. Two of the students, which represents 8% of the students, in Ms. Clark's class wore red shirts today. How many students are in Ms. Clark's class?

Simplify the expressions with exponents below. (DOK 2)

1. $15 - 2^2$

2. $6^2 + 13$

3. 8×5^2

4. $35 - 3^3$

5. $9^2 + 12$

6. $2^4 \div 4$

7. $5^3 - 38$

8. $18 + 3^2$

9. $4^3 \div 8$

10. $19 - 2^2$

11. $75 \div 5^2$

12. $10^2 + 114$

13. $8^2 \times 3$

14. $99 - 6^2$

15. $4^2 \times 7$

16. $17 - 3^2$

17. $7^2 - 27$

18. $11 + 5^3$

19. $12^2 \div 9$

20. $23 - 4^2$

21. $184 \div 2^2$

22. $6^2 + 40$

23. $11^2 - 38$

24. $2^5 + 12$

Basic Properties of Rational Numbers (DOK 2)

The **Associative**, **Commutative**, and **Distributive** properties and the **Identity** and **Inverse** properties of addition and multiplication are listed below by example as a quick refresher.

Property	Example
1. Associative Property of Addition	$(a + b) + c = a + (b + c)$
2. Associative Property of Multiplication	$(a \times b) \times c = a \times (b \times c)$
3. Commutative Property of Addition	$a + b = b + a$
4. Commutative Property of Multiplication	$a \times b = b \times a$
5. Distributive Property	$a \times (b + c) = (a \times b) + (a \times c)$
6. Identity Property of Addition	$0 + a = a$
7. Identity Property of Multiplication	$1 \times a = a$
8. Inverse Property of Addition	$a + (-a) = 0$
9. Inverse Property of Multiplication	$a \times \dfrac{1}{a} = \dfrac{a}{a} = 1, a \neq 0$

Equivalent Expression Practice (DOK 2)

1. Which expressions are equivalent to $2(x + 8)$?

   A. $2x + 8$

   B. $2x + 16$

   C. $2x + 4 \times 2 \times 2$

   D. $3(x + 8) - (x + 8)$

   E. $3(x + 8) - (x - 8)$

2. Which expressions are equivalent to $4(x - 5)$?

   A. $4x - 20$

   B. $4x - 5$

   C. $(2x - 2) + (2x - 3)$

   D. $4x + 3 - 8$

   E. $4 + x - 5$

3. Which expressions represent the sum of 6 and x?

   A. $6x$

   B. $6 + x$

   C. $x + 6$

   D. $x + x + x + x + x + x$

   E. x^6

4. Which expressions represent the difference of x^3 and $2x$?

   A. $x^3 - 2x$

   B. $x \times x \times x - 2x$

   C. $3x - 2x$

   D. $x + x + x - x + x$

   E. $-2x + x^3$

Select each expression that is equivalent.

5. $4(2x + 5)$

   A. $8x + 5$

   B. $8x + 20$

   C. $4x + 2 + 4x + 3$

   D. $3(3x + 5) - (x - 5)$

   E. $3(2x + 5) + (2x + 5)$

6. $5(x - 4)$

   A. $5x - 4$

   B. $5x - 20$

   C. $4x - 4 + x - 16$

   D. $5x + 1$

   E. $2(x + 1) + (3x - 7)$

7. $5x + 2 - 3x - 1$

   A. $2(x + \frac{1}{2})$

   B. $2(x + 1)$

   C. $2(x - 1)$

   D. $4(x + 1) - 3(x + 1)$

   E. $7x - 4x$

8. $4(x + 6) - (x + 6)$

   A. $3(x + 6)$

   B. $x + x + x + 18$

   C. $4x + 24 - x + 6$

   D. $3(x - 6)$

   E. $3x + 18$

Chapter 8
Introduction
to Algebra

This chapter covers the following standards: 6.EE.2, 6.EE.4, 6.EE.6

Algebra Vocabulary (DOK 2)

Algebra is the branch of mathematics that uses a combination of letters, numbers, and operations (addition, subtraction, multiplication, and division) to show how two or more things are related to each other. Solving algebra problems can be made simple if you learn the language and rules. Algebra can also be used to solve problems that arise in real life. The table below shows the basic vocabulary for solving algebra problems.

Vocabulary Word	Example	Definition
expression	$4x + 3$	a mathematical combination of numbers, variables, and operations
variable	$4x$ (x is the variable)	a letter that can be replaced by a number
coefficient	$4x$ (4 is the coefficient)	a number multiplied by a variable or variables
term	$5x^2 + x - 2$ ($5x^2$, x, and 2 are terms)	numbers or variables separated by $+$ or $-$ signs
constant	$5x + 2y + 4$ (4 is a constant)	a term that does not have a variable
sentence	$2x = 7$ or $5 \leq x$	two algebraic expressions connected by $=$, $\neq, <, >, \leq$ or $\geq$
equation/equality	$4x = 8$	a sentence with an equal sign
inequality	$7x < 30$ or $x \neq 6$	a sentence with one of the following signs: $\neq, <, >, \leq, \geq$, or $=$
solution	if $3x = 9$, then $x = 3$	numbers that will make a sentence true
base	6^3 (6 is the base)	the number used as a factor
exponent	6^3 (3 is the exponent)	the number of times the base is multiplied by itself

Substituting Numbers in Formulas (DOK 2)

Many formulas are used in mathematics. There are formulas for finding the surface area of an object, the volume of an object, the rate of speed of a vehicle, etc. When describing the measures used in a formula, follow these rules:

The variable x^2 is read x squared.

> **Example:** Stacey's paper has an area of 68 in^2.

The variable x^3 is read x cubed.

> **Example:** Robert's cylinder has a volume of 122 in^3.

For each of the following expressions, substitute $a = 4$, $b = 6$, $c = 3$ and $d = 5$ and find the value of the expression.

1. $b^2 - 2a \div 4 + 2d$
2. $d^2 - c^2 + 4b - 3a$
3. $2(d - c) + 7a$

4. $5b \div c \times a + 3d$
5. $2a^2 + 4b \div c + d^2$
6. $cd + ab - c^2 + 10$

7. $c^2 + 3a \div b - 2d$
8. $4c^2 + 4b \div a - 5d$
9. $3(d^2 - c) \div 3 + 2b$

In the problems below, a formula and the value of the variable in the formula will be given. Substitute the value of the variable in each formula and solve. The first one is done for you. (DOK 2)

10. The formula for finding the volume of a cube is $V = s^3$, where s is the length of the side of a cube. Terrance has a cube that measures 3 inches along each side. What is the volume of Terrance's cube?

    $V = 3^3 = 27$ in^3.

11. The formula for the area of a rectangular piece of paper is $A = lw$. The length, l, of Lizetta's piece of paper is 6 inches. The width, w, of her paper is 4 inches. What is the area of Lizetta's piece of paper in square inches?

12. The formula for the volume of a rectangular prism is $V = lwh$. Maurice has a box that measures $l = 14$ inches, $w = 6$ inches, and $h = 4$ inches. What is the volume of Maurice's box? Express your answer in cubic inches.

13. The formula for the surface area of a cube is $s^2 \times 6$. Nicole has a cube that measures 10 centimeters on each edge. What is the volume of Nicole's cube? Express your answer in square centimeters.

14. The formula for the area of a rectangular piece of cardboard is $A = lw$. The length, l, of Elijah's piece of cardboard is 25 cm. The width, w, of his cardboard is 10 cm. What is the area of Elijah's piece of cardboard in square cm?

15. The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. Grace has a pillow shaped like a pyramid. The base is 144 square inches. The height is 10 inches. What is the volume of Grace's pillow shaped like a pyramid in cubic inches?

Example 2:   Solve for k: $\frac{k}{8} = 10$

In this equation, $\frac{k}{8}$ shows division; so multiplication, or the opposite operation, must be done to solve the equation. Both sides must be multiplied by 8.

$8 \times \frac{k}{8} = 8(10)$

$k = 80$

To check, replace the answer, 80, for k in the original problem. $\frac{80}{8} = 10$
80 is the number that makes this sentence true.

Use multiplication and division to solve the equations below. Show your work. (DOK 2)

1. $9m = 81$

2. $3k = 27$

3. $\frac{y}{2} = 166$

4. $6t = 120$

5. $7m = 112$

6. $\frac{x}{5} = 11$

7. $\frac{q}{15} = 4$

8. $13m = 52$

9. $8a = 32$

10. $27b = 81$

11. $15x = 615$

12. $6x = 48$

13. $\frac{h}{60} = 2$

14. $\frac{b}{2} = 167$

15. $\frac{z}{15} = 3$

16. $3n = 36$

17. $4k = 64$

18. $48x = 96$

19. $\frac{w}{5} = 20$

20. $\frac{x}{3} = 15$

Solving Algebra Word Problems (DOK 2, 3)

One-Step Word Problems

Just like solving any word problem, the key is to READ CAREFULLY. Use the information given to solve what the question is asking.

Example: Mrs. Hill bought 3 pounds of apples. The price is $2.49 per pound. How much did Mrs. Hill pay for the 3 pounds of apples?

Step 1:   Get the facts together: 3 pounds of apples that cost $2.49 per pound. Multiply $3 \times \$2.49 = \$7.47$

Answer: $7.47

Example 2: Write an equation that can be used to find y, the amount spent on d, dozen donuts.

Cost of Donuts	
Dozen	*Cost*
1	1.80
2	3.60
3	5.40
4	7.20

Answer: $y = \$1.80d$

For each of the following, write the equation to find y, the total cost of x number of items. (On some problems, you may need to find the cost of one item first.)

1.

Boxes of Cereal	Total Cost
1	3.49
2	6.98
3	10.47

How much do six boxes of cereal cost?

2.

Pairs of Socks	Total Cost
2	1.50
4	3.00
6	4.50

How much do five pairs of socks cost?

3.

Mary's Dairy	
Gallons of Milk	Price
2	7.58
4	14.16

How much does six gallons of milk cost?

4.

Reruns	
Number of Shirts	Price
3	12.00
8	32.00

How much did Mrs. Miller pay for five shirts?

5. Jeff earns a 20% commission on his sales. Last week he sold $80,000 worth of product, p. Find his total commission, t, using this equation:
$$.20p = t$$

6. Mike gets a $4.25 allowance each week. He uses this equation to figure his total allowance, t, for w weeks.
$$t = 4.25w$$
How much allowance does he earn in 15 weeks?

7. Mr. Allen needs seven gallons of paint for his house. It is on sale at three gallons for $54.78. Using the equation where t is total cost, what is the price of seven gallons of paint, g?
$$t = \$18.26g$$

8. Pat needs to earn more money to buy a game, g. His savings, s, is $8.40. He needs $46.20 total. How much more does he need to buy the game? Use his equation.
$$g = \$46.20 - s$$

Write the inequality represented by each of the following number lines. (DOK 2)

11. _____

15. _____

12. _____

16. _____

13. _____

17. _____

14. _____

18. _____
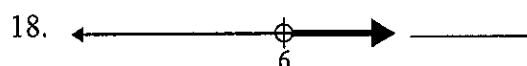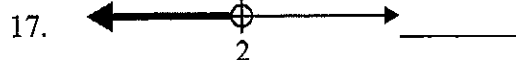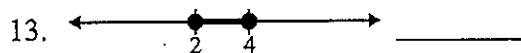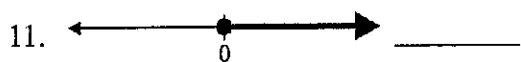
Solving Inequalities by Addition and Subtraction (DOK 2)

Solving inequalities is similar to solving equations.

Example: Solve and graph the solution set for $x - 2 \leq 5$.

Step 1:   Add 2 to both sides of the inequality, so the variable will be by itself.

$$x - 2 \leq 5$$
$$\underline{+2 \quad\; +2}$$
$$x \;\; \leq 7$$

Step 2:   Graph the solution set for the inequality.

Solve and graph the solution set for the following inequalities. (DOK 2)

1. $x - 1 > 1$

2. $x - 10 < 5$

3. $x - 2 \leq 1$

4. $8 + x \geq 15$

5. $x + 2 > 4$

6. $x + 11 \leq 20$

7. $2 + x > 5$

8. $x + 7 \leq 12$

9. $x + 6 \geq 7$

10. $15 + x > 25$

11. $x + 6 < 10$

12. $3 + x > 7$

13. $x - 2 > 4$ ⟵─────────⟶   17. $x - 3 < 17$ ⟵─────────⟶

14. $x - 8 > 24$ ⟵─────────⟶   18. $x + 7 < 17$ ⟵─────────⟶

15. $x + 1 \leq 12$ ⟵─────────⟶   19. $x + 6 \leq 14$ ⟵─────────⟶

16. $11 + x \geq 11$ ⟵─────────⟶   20. $x - 8 \geq 19$ ⟵─────────⟶

Solving Inequalities by Multiplication and Division (DOK 2)

Example: Solve and graph the solution set for $4x \leq 20$.

Step 1:   Divide both sides of the inequality by 4. $\dfrac{\overset{1}{\cancel{4}}x}{\underset{1}{\cancel{4}}} \leq \dfrac{\overset{5}{\cancel{20}}}{\underset{1}{\cancel{4}}}$

(Note: A fraction bar also means division.)

Step 2:   Graph the solution. $x \leq 5$ ⟵━━━━━━━●────⟶
$\boxed{0}$ $\boxed{5}$

Solve and graph the following inequalities. (DOK 2)

1. $\dfrac{x}{5} > 4$ ⟵─────⟶   9. $9x \leq 54$ ⟵─────⟶

2. $2x \leq 24$ ⟵─────⟶   10. $\dfrac{x}{8} > 1$ ⟵─────⟶

3. $6x > 24$ ⟵─────⟶   11. $\dfrac{x}{9} \geq 3$ ⟵─────⟶

4. $3x \leq 33$ ⟵─────⟶   12. $4x < 12$ ⟵─────⟶

5. $\dfrac{x}{5} > 2$ ⟵─────⟶   13. $\dfrac{x}{2} \leq 20$ ⟵─────⟶

6. $7x \leq 49$ ⟵─────⟶   14. $10x \leq 30$ ⟵─────⟶

7. $2x \leq 16$ ⟵─────⟶   15. $\dfrac{x}{5} < 4$ ⟵─────⟶

8. $\dfrac{x}{8} > 4$ ⟵─────⟶   16. $5x > 30$ ⟵─────⟶

Making a Table of Solutions From an Equation (DOK 2)

Example 1:   Given the equation $y = 2x$, make a table of solutions that match the equation.

Step 1:   Write any values for x. We chose 0, 1, 2, and 3.

$y = 2x$	
x	y
0	
1	
2	
3	

Step 2:   Plug each value for x into the equation to find the value for y.
$y = 2(0)$; $2 \times 0 = 0$, so $y = 0$
$y = 2(1)$; $2 \times 1 = 2$, so $y = 2$
$y = 2(2)$; $2 \times 2 = 4$, so $y = 4$
$y = 2(3)$; $2 \times 3 = 6$, so $y = 6$

$y = 2x$	
x	y
0	0
1	2
2	4
3	6

Example 2:   Mr. Sloan travels in the city at an average rate of speed of 42 miles per hour. Set up a table to show how long will it take him to go anywhere.

Using the equation $d = 42t$ where d is distance and t is time in hours, fill in different values for t and d.

$d = 42t$	
t	d
1	42
2	84
3	126
4	168

Find the values of the variables in the tables below. The situations and equations have been set up for you. (DOK 2)

1. Sophia takes 22 minutes to read 10 pages. If she reads at the same rate, how long will it take Sophia to read books that are 200 pages, 220 pages, 240 pages, and 260 pages long? The equation shows x equals the number of pages, and y equals the number of minutes.

$y = \dfrac{x}{10} \times 22$	
x	y
200	440
220	
240	
260	

2. Jose takes 5 minutes to do 3 math problems. If he does the equations at the same rate, how long will it take Jose to do 9 problems, 12 problems, 15 problems, or 18 problems? The equation shows x equals the number of problems, and y equals the number of minutes.
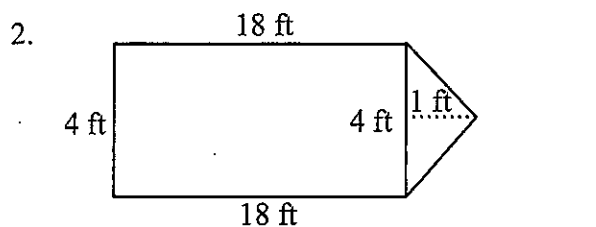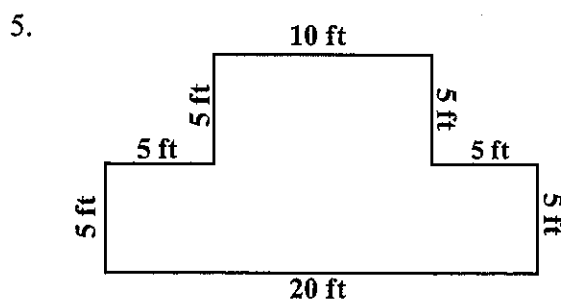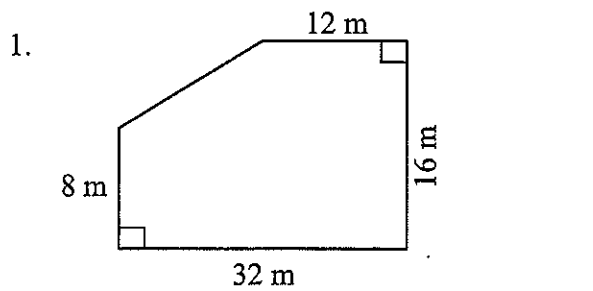
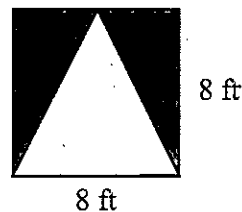| $y = \dfrac{x}{3} \times 5$	
x	y
9	15
12	
15	
18	

7 yd

6 yd

$7 \times 6 = 42 \text{ yd}^2$

Step 3:   Subtract the area of the shaded part from the area of the complete rectangle.
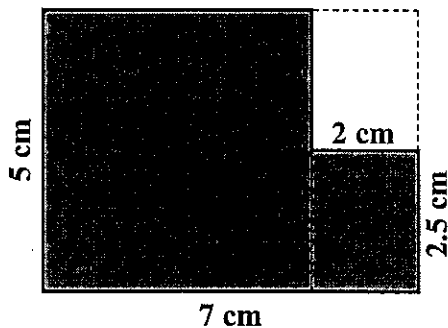
$252 - 42 = 210 \text{ yd}^2$

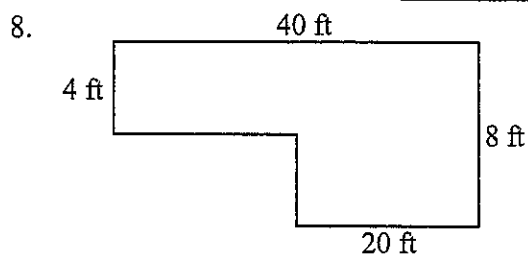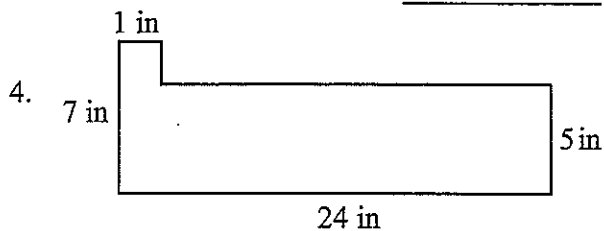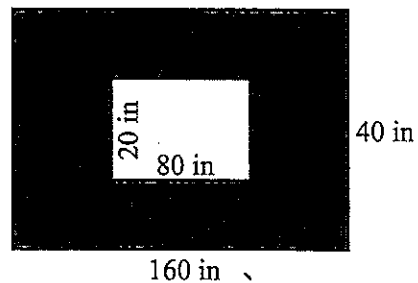**Find the area of the following figures. (Figures are not drawn to scale.) (DOK 3)**

1.

12 m

8 m

16 m

32 m

5.

10 ft

5 ft

5 ft

5 ft

5 ft

5 ft

5 ft

5 ft

20 ft

2.

18 ft

4 ft

4 ft

1 ft

18 ft

6.   What is the area of the shaded part?
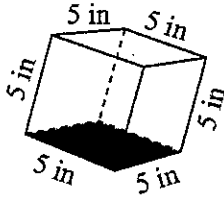
8 ft

8 ft

3.   What is the area of the shaded part?

5 cm

2 cm

2.5 cm

7 cm

7.   What is the area of the shaded part?

20 in

80 in

40 in

160 in

4.

1 in

7 in

5 in

24 in

8.

40 ft

4 ft

8 ft

20 ft

Chapter 12 Review Test

**Find the volume and/or surface area of the following solids. Formulas are provided. (DOK 2)**

1.

$V_{cube} = s^3$

$SA_{cube} =$ area of all the faces, $s^2 \times 6$

2. The sandbox at the local elementary school is 80 inches wide and 120 inches long. The sand in the box is 4 inches deep. How many cubic inches of sand are in the sandbox when it is full? $V_{rectangular\ prism} = lwh$

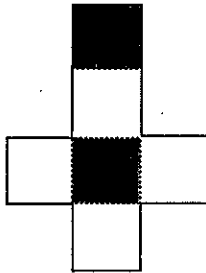**3.** Find the surface area of the triangular prism below.
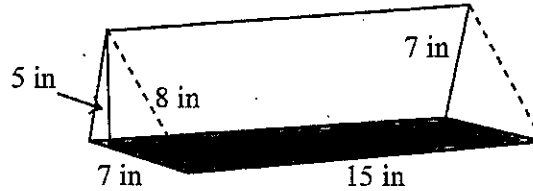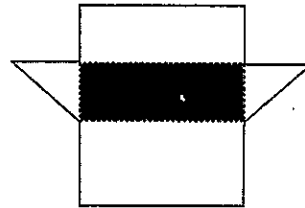

$SA = 2B + PH, B = \frac{1}{2}bh$

4. Siena wants to build a wooden toy box with a lid. The dimensions of the toy box are 2 feet long, 5 feet wide, and 1 foot tall. How many square feet of wood will she need to construct the box? $SA_{rectangular\ prism} =$ sum of the area of all the faces

Name the solid figure represented by the nets below. (DOK 1)

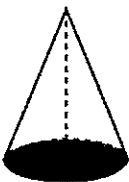**5.**

6.

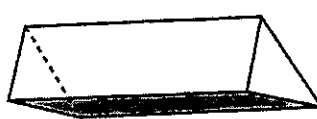**Answer the following questions. (DOK 1)**

7. What three-dimensional figure has exactly one vertex?

8. What three-dimensional figure has exactly four vertices?

9. How many faces does a cylinder have?

10. How many edges does a rectangular prism have?

Name the three-dimensional figures below. (DOK 1)
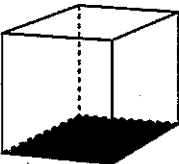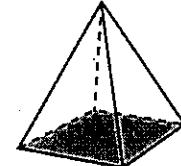
11.

12.

13.

14.

15.

16.

Mean (DOK 1, 2)

The **mean** of a set of numbers is also known as the average or center of the set. Average is more often used to describe real life situations. To find the mean or average of a set of data, add the numbers in the list, and then divide by the total number of items on the list.

Don't forget ZERO! Zero, 0, may be a number in the list. Although zero has "no" value, it must be included in the addition step and counted as a number on the list.

Example 1:	Clay had the following work hours for the week: Monday 5 hours, Tuesday 6 hours, Wednesday 0 hours, Thursday 4 hours, Friday 0 hours, Saturday 6 hours, and Sunday 0 hours. What was the mean (or average) number of hours?
Step 1:	Find the total number of hours he worked by adding all of the hours together. $5 + 6 + 0 + 4 + 0 + 6 + 0 = 21$
Step 2:	Divide the total number by the number of items in the set. The number of items is 7 (the number of days). 21 divided by 7 is 3. **Clay's mean number of hours was 3.**
Example 2:	Mr. Jones' class scored the following grades on its science fair projects: 98, 96, 96, 94, 92, 88, 82, 80, 76, 74, 74, 72, 68, 68, and 60. What was the mean test score?
Step 1:	Add up all of the grades. $98+96+96+94+92+88+82+80+76+74+74+72+68+68+60 = 1218$
Step 2:	Divide the total of the grades by the number of grades in the list. There are 15 grades, so divide 1218 by 15. **The mean test score is 81.2.**

Find the mean for each set. Round all answers to the nearest tenth. (DOK 1)

1. 65, 70, 80, 90, 95
2. 16, 12, 8, 4, 0, 0, 10, 28, 14
3. 5, 11, 7, 9, 3
4. 4, 5, 2, 4, 5
5. 3, 6, 9, 2, 10

6. 75, 65, 85, 70, 80
7. 100, 90, 60, 0, 80, 80, 75, 85, 90, 90
8. 4.5, 6.2, 4.2, 3.2, 3.8, 4.2, 4.6, 5.4, 6.8, 2.2
9. 489, 560, 423, 550
10. $45, $50, $60, $75, $45

Find the mean asked in each word problem below. (DOK 2)

11. Dolly's Deli keeps track of all the restaurant's sales. Here are the sales for chicken salad sandwiches for one week: 65, 76, 80, 50, 30, 88, and 50. What was the average or mean number of chicken salad sandwiches sold?

12. Dave's office tracks his incoming calls. Here is the record for his calls for this week: 16, 14, 12, 10, and 8. What was the average number of calls?

13. Tina earned $60 baby-sitting over the weekend for a total of 5 hours. What was her average rate per hour?

14. Maddie scored 18 RBIs in 10 games. What was her average RBI count per game?

15. Aaliyah drove 488 miles in 8 hours. What was her average speed?

Mean Absolute Deviation (DOK 3)

Variability is the measure of differences of things, such as scores and temperatures, with the mean(average) as the reference point. The **mean absolute deviation** is the average distance between all elements in a data set and the mean of the data set.

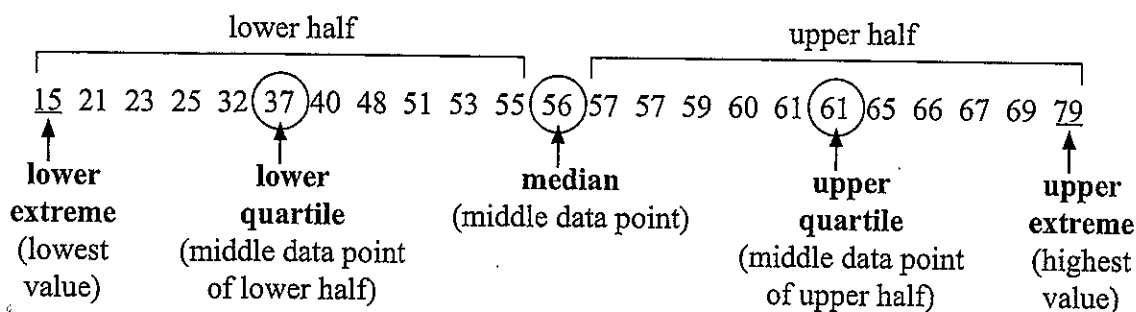| **Example 1:**	In the data set 1, 1, 4, 7, and 12, the mean is 5. However, the mean absolute deviation is 3.6. This concludes that there is an average of 3.6 units between the elements in the set and the mean. Although you will not be required to figure the mean absolute deviation yourselves, the chart to the right shows how the mean absolute deviation was calculated.	**Mean Absolute Deviation**	
		$\|5-1\|$	$\|4\|$
		$\|5-1\|$	$\|4\|$
		$\|5-4\|$	$\|1\|$
		$\|5-7\|$	$\|-2\|$
		$\|5-12\|$	$\|-7\|$
		Sum:	$\|18\|$
		Mean:	$\|18\| \div 5 = 3.6$

In statistics, large sets of data are separated into four equal parts. These parts are called **quartiles**. The **median** separates the data into two halves. Then, the median of the upper half is the **upper quartile**, and the median of the lower half is the **lower quartile**. The distance between the upper quartile and lower quartile is called the **interquartile range**.

The **extremes** are the highest and lowest values in a set of data. The lowest value is called the **lower extreme**, and the highest value is called the **upper extreme**.

Example 2: The following set of data shows the high temperatures (in degrees Fahrenheit) in cities across the United States on a particular autumn day. Find the lower extreme, the lower quartile, the median, the upper quartile, and the upper extreme of the data. Then, find the interquartile range of the data.

lower half   upper half

15 21 23 25 32 (37) 40 48 51 53 55 (56) 57 57 59 60 61 (61) 65 66 67 69 79

lower extreme (lowest value)   **lower quartile** (middle data point of lower half)   **median** (middle data point)   **upper quartile** (middle data point of upper half)   **upper extreme** (highest value)

The lower extreme is 15, the lower quartile is 37, the median is 56, the upper quartile is 61, and the upper extreme is 79. Finally, the interquartile range is 24. The **interquartile range** is the difference between the upper and lower quartiles.

To briefly review, the **mean absolute deviation** is the average distance between any elements in a data set and the mean of the data set. And the **interquartile range** is the distance between the upper and lower quartiles of the elements in a data set.

Example 3:   Sandra conducted an experiment by randomly asking two sets of 7 students on how old they were.

The two sets of data are:

Set 1:	11, 12, 12, 13, 14, 14, 15
Set 2:	10, 11, 12, 12, 14, 15, 17

Discuss the variability of the mean absolute deviation and the interquartile range between the two sets of data.

		Mean Absolute Deviation	**Interquartile Range**
Given:	**Set 1:**	1.143	2
	Set 2:	2	4

Discussion: Without looking at the data sets, we can see by the mean absolute deviation of the two sets of data, that there is a wider variability of ages in set 2. We can make the same conclusion looking at the interquartile range.

Fact: Interestingly enough, both sets of data have the same mean of 13.

Students in Mrs. Bearin's class were asked to take two sets of data of the ages of people in different populations, such as their neighborhoods, church, grocery store, extended family, etc. Five of the students' data sets collected are shown below. The mean absolute deviation and the interquartile range of each two sets of data are given. Draw conclusions based on the variability of the data given (on how the data varies from one set to the other) similar to Example 3 above. **(DOK 3)**

1.

	Mean Absolute Deviation	**Interquartile Range**
Set 1:	4.2	12
Set 2:	1.3	6

2.

	Mean Absolute Deviation	**Interquartile Range**
Set 1:	7.62	17
Set 2:	11.47	35

3.

	Mean Absolute Deviation	**Interquartile Range**
Set 1:	0.5	2
Set 2:	8.2	23

4.

	Mean Absolute Deviation	**Interquartile Range**
Set 1:	9.7	44
Set 2:	2.5	8

5.

	Mean Absolute Deviation	**Interquartile Range**
Set 1:	14.33	107
Set 2:	4.41	2

Variability (DOK 1)

Questions or statements with more than one answer are said to have **variability**. For instance, if you are asked how old you are, there is only one answer. But if you are asked how old <u>all</u> of the students in your class are, the answers will vary. The question expects variability. A statement saying a person has one pet, does not show variability. However, a statement saying a person has one pet, and another person has three pets, does show variability.

Make a check mark on the line in front of the questions that expect variability. (DOK 1)

1. _____ How old are each of the dogs in this store?

2. _____ Larry's brother is 16 years old.

3. _____ Mrs. Andrews asked for a show of hands and found 24 students from one 6th grade class and 27 students from another 6th grade class are going on the field trip.

4. _____ How many people view the movie at each showing at the theatre?

5. _____ There are 14 fish in the fish tank.

6. _____ Janine rode the bus to school four days this week.

7. _____ The school records show that 1,102 salads were served Monday, and 983 salads were served Wednesday.

8. _____ How many students in your class are over 5 feet tall?

9. _____ A survey of Mr. Bryant's class shows that 6 students have no sibling, 4 students have 1 sibling, and 14 students have 2 or more siblings.

10. _____ What was the attendance rate each school day last month?

11. _____ The average weight of a hamster in the Happy Pet Store is under 3 ounces.

12. _____ Harold jogged 3 miles today.

13. _____ How many kids are in each house or apartment in your neighborhood?

14. _____ A stadium ticket office sold 6,752 tickets for Friday's game and 11,836 tickets for Saturday's game.